High-Dimensionality in data and their projections

DSTA

Data Science context

- Dataset: n points in a d-dimensional space:
- essentially, a $n \times d$ matrix of floats
- For $d > 3$ and growing, several practical problems

1-hot encodings raise dimensionality

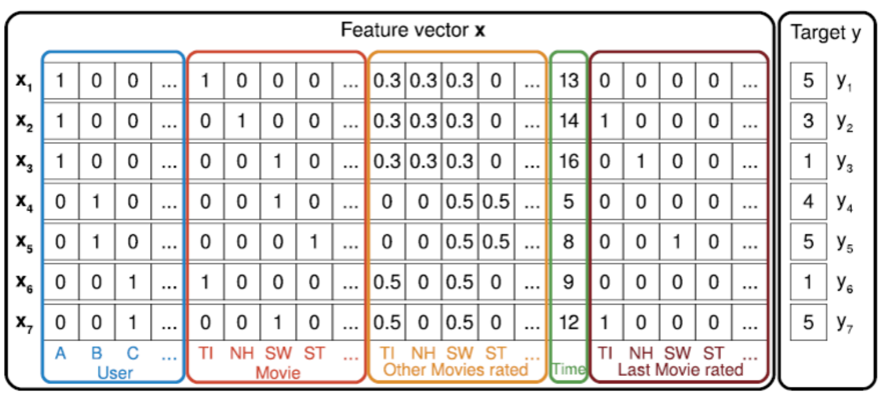

Fig. 1. Example (from Rendle [2010]) for representing a recommender problem with real valued feature vectors $\mathbf{x}$. Every row represents a feature vector $\mathbf{x}_i$ with its corresponding target y_i. For easier interpretation, the features are grouped into indicators for the active user (blue), active item (red), other movies rated by the same user (orange), the time in months (green), and the last movie rated (brown).

How to see dimensions

data points are row vectors

X1

X2

...

Xd

x1

x11

x12

...

x1d

...

...

...

...

...

xn

xn1

xn2

...

xnd

Issues

- visualization is hard, we need projection. Which?
- decision-making is impaired by the need of chosing which dimensions to operate on
- **sensitivity analyis** or causal analysis: which dimension affects others?

Issues with High-Dim. data

I: a false sense of sparsity

adding dimensions makes points seems further apart:

Name

Type

Degrees

Chianti

Red

12.5

Grenache

Rose

12

Bordeaux

Red

12.5

Cannonau

Red

13.5

d(Chianti, Bordeaux) = 0

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let type differences count for 1:

d(red, rose) = 1

take the alcohol strengh as integer tenths-of-degree: d(12, 12.5) = 5

. . .

d(Chianti, Grenache) = $\sqrt{1^2 + 5^2} = 5.1$

Adding further dimensions make points seem further from each other

not close anymore?

Name

Type

Degrees

Grape

Year

Chianti

Red

12.5

Sangiovese

2016

Grenache

Rose

12

Grenache

2011

Bordeaux

Red

12.5

2009

Cannonau

Red

13.5

Grenache

2015

d(Chianti, Bordeaux) > 7

d(Chianti, Grenache) $> \sqrt{5^2 + 1^2 + 5^2} = 7.14$

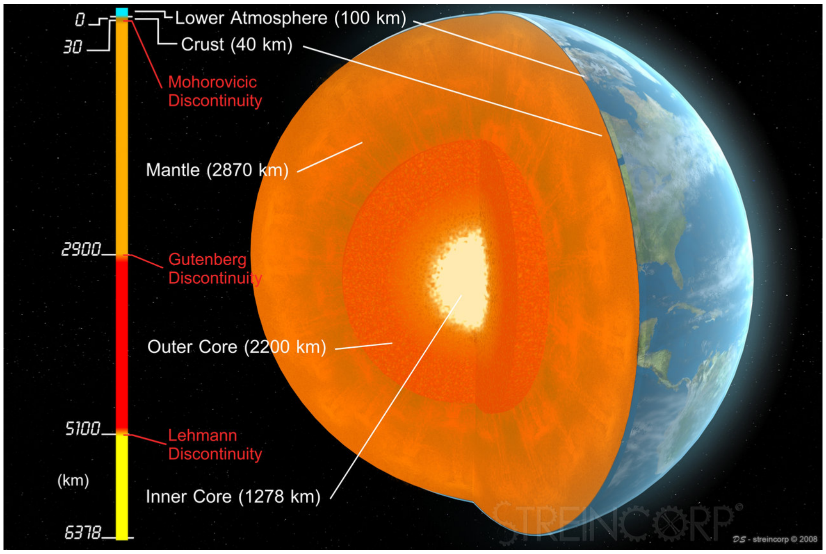
II: the collapsing on the surface

Bodies have most of their mass distributed close to the surface (even under uniform density)

the *outer* orange is twice as big, but how much more juice will it give?

- for d=3, $vol = \frac{4}{3}\pi r^3$.
- With 50% radius, vol. is only $\frac{1}{8} = 12.5\%$

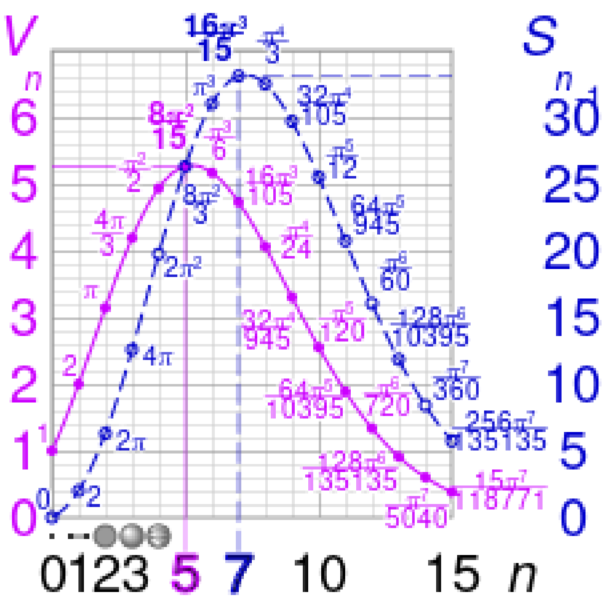
Possibly misguiding



The most volume (and thus weight) is in the external ring (the equators)

counter-intuitive properties

At a fixed radius (r=1), raising dimensionality *above 5* in fact decreases the volume.



Hyperballs deflate.

Geometry is not what we experienced in $d \leq 3$.

The Curse of dimensionality

Volume will concentrate near the surface: most points will look as if they are at a uniform distance from each other

- distance-based similarity fails

Consequences

Adding dimensions apparently increases sparsity

Deceiving as a chance to get a clean-cut segmentation of the data, as we did with Iris

. . .

In high dimension, all points tend to be at the same distance from each other

Exp: generate a set of random points in D^n, compute Frobenius norms: very little variance.

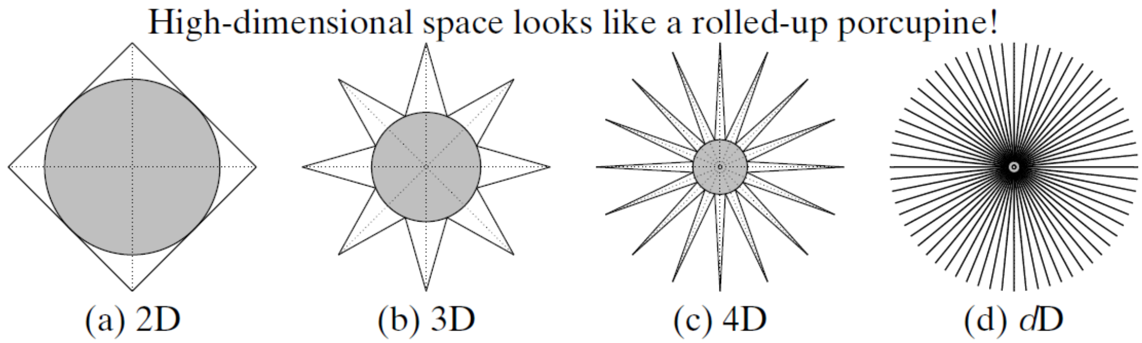
. . .

bye bye clustering algorithms, e.g., k-NN.

The porcupine

At high dimensions,

- all diagonals strangely become orthogonal to the axes
- points distributed along a diagonal gets *"compressed down''* to the origin of axes.



. . .

bye bye to all distance-based algorithms and similarity measures, e.g., Cosine Similarity.

Where are all my data points?