Financial networks

DSTA

1 Financial Networks

1.1 Introduction

Theme: discover a relationship among traded shares (equity)

. . .

look at historical market data to see whether price variations relate to each other.

Are there *regularities* that could anticipate the future behaviour of price?

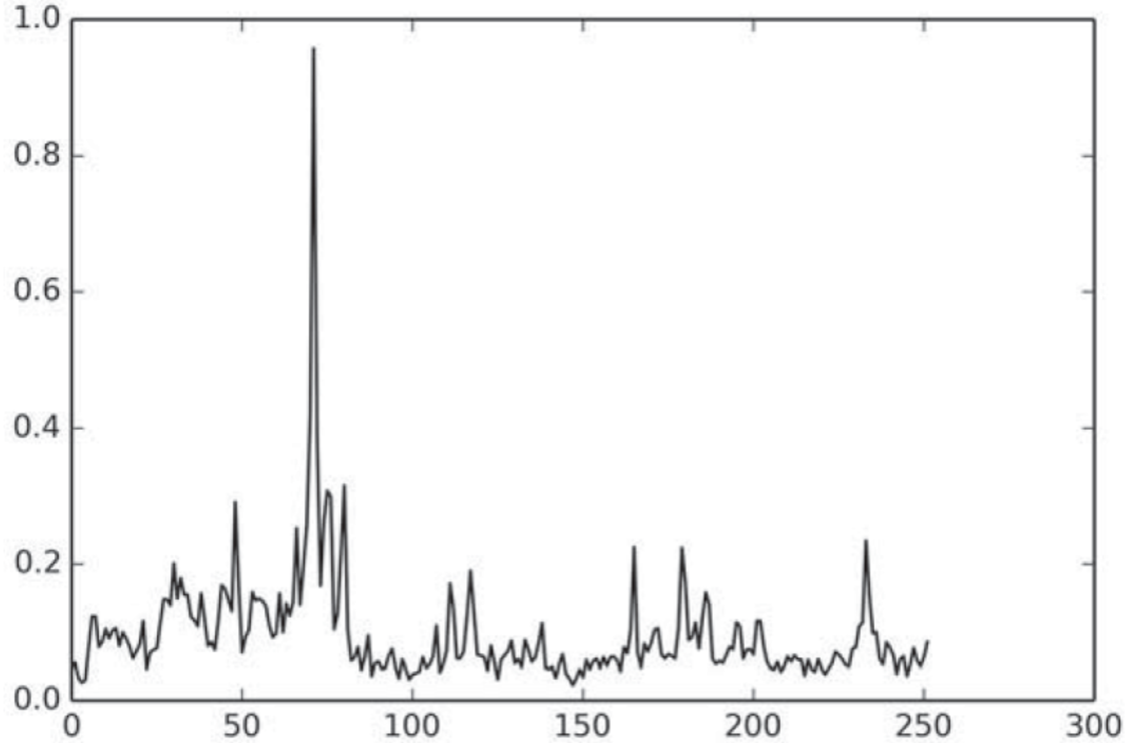
. . .

In Food Networks (Ch. 1) we discovered a regularity:

$\frac{\#pred}{\#prey} \approx 1$

1.2 Important assumption

When markets are *calm*, investment becomes somewhat "mathematical' '



2 Price time series

2.1 Proportional return on investment

- depends on time
- essentially, the discrete counterpart of the time derivative of price:

$$r(\Delta t) = \frac{p(t_0 + \Delta t) - p(t_0)}{p(t_0)}$$

$$r(\Delta t) = \frac{p(t_0 - \Delta t) - p(t_0)}{p(t_0)}$$

. . .

in the limit $\Delta t \to 0$ it can be rewritten:

$$r(t) \simeq \frac{d \ln p(t)}{dt}$$

. . .

For discrete time:

$$r = \ln p(t_0 + \Delta t) - \ln p(t_0)$$

2.2 Correlation of prices

- correlations in time series (or simply *comovements*) are valuable indicators
- Two shares are correlated if historically their price varied *in a similar way.*
- To qualify such a relation compute the correlation between their price returns over Δt.

. . .

Let $\langle r_i \rangle$ be the average return of i over Δt

$$\rho_{ij}(\Delta t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{(\langle r_i^2 \rangle - \langle r_i \rangle^2)(\langle r_j^2 \rangle - \langle r_j \rangle^2)}}$$

- high ρ's might uncover hidden links between stocks.
- however, monitoring $n(n-1)$ correlations quickly becomes unfeseable
- we focus on high ρ values.

3 The Spanning tree of stocks

3.1 Similar-behaviour shares

Correlation (or lack of it) induces a *distance* b/w shares:

$$d_{ij}(\Delta t) = \sqrt{2(1 - \rho_{ij}(\Delta t))}$$

Let $D(\Delta t)$ be the complete matrix of pairwise distances:

it describes a complete, weighted network!

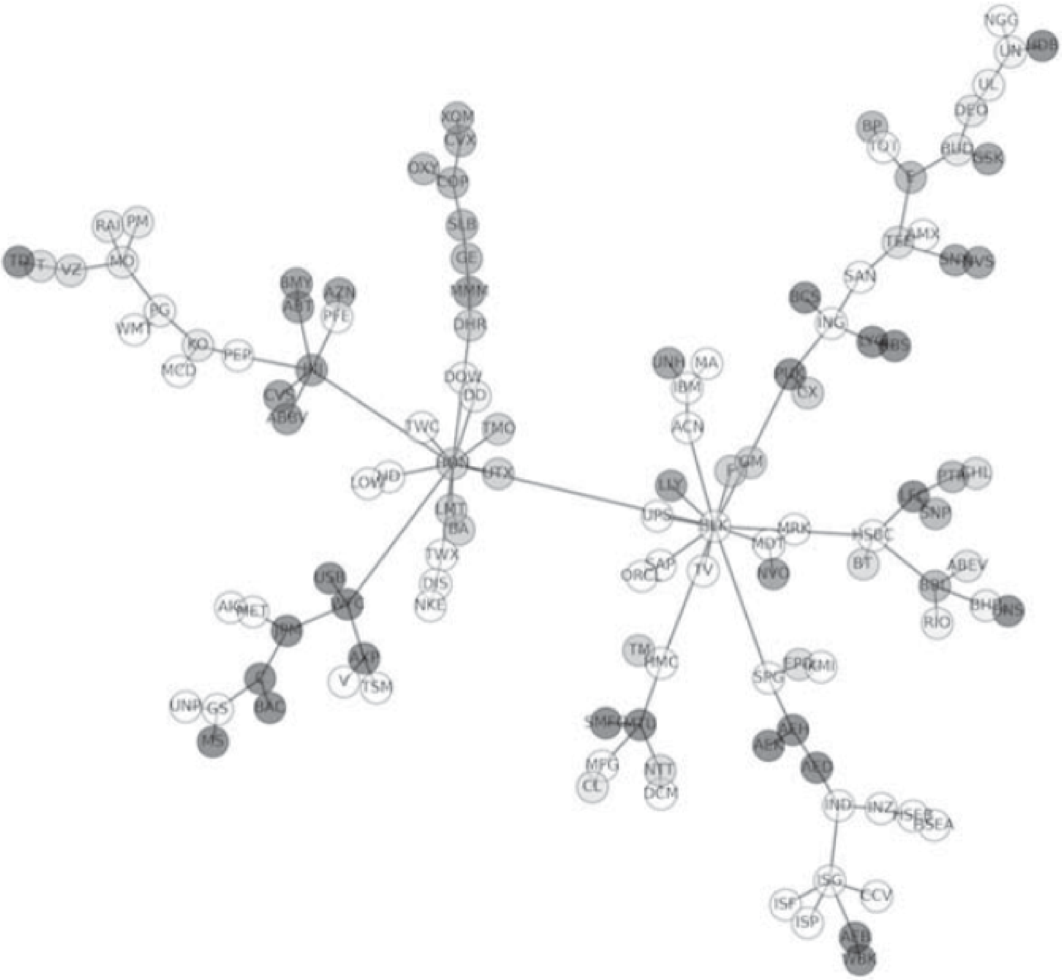
. . .

Prune it to create its Mimimum Spanning Tree (MST)

The MST has only n-1 *heavy* connections while maintaining connectivity.

3.2 Resulting model

The MST of 141 NYSE high-cap stocks, Δt = 6h:30min



Some shares are *hubs* for local clusters of highly-correlated shares.

3.3 Consequences

- Network analysis helps indentifying local clusters
- Each clusters will have a “hub” share
- Hub shares can *signal* the beaviour of the whole cluster:
- they provide leads in forecasting how sections of the market will move.